

Opener

Calculator

The graph of the function $y = x^3 + 6x^2 + 7x - 2\cos x$ changes concavity at $x =$

- (A) -1.58 (B) -1.63 (C) -1.67 (D) -1.89 (E) -2.33

Non-Calculator

The graph of $y = 3x^4 - 16x^3 + 24x^2 + 48$ is concave down for

- (A) $x < 0$
 (B) $x > 0$
 (C) $x < -2$ or $x > -\frac{2}{3}$
 (D) $x < \frac{2}{3}$ or $x > 2$
 (E) $\frac{2}{3} < x < 2$

$$y' = 3x^2 + 12x + 7 + 2\sin x$$

$$y'' = 6x + 12 + 2\cos x$$

4-3 day 3 The Second Derivative Test for Local Extrema

Learning Objectives:

I can use the second derivative test to classify local extrema of a function.

Exploration #2 on pg 214

EXPLORATION 2 Finding f from f' and f''

A function f is continuous on its domain $[-2, 4]$, $f(-2) = 5$, $f(4) = 1$ and f' and f'' have the following properties.

x	$-2 < x < 0$	$x = 0$	$0 < x < 2$	$x = 2$	$2 < x < 4$
f'	+	does not exist	-	0	-
f''	+	does not exist	+	0	-

Handwritten notes:
 - Above the first column: *inc & ccu*
 - Above the second column: *dec ccv*
 - Above the third column: *dec ccv*

1. Find where all absolute extrema of f occur.
2. Find where the points of inflection of f occur.
3. Sketch a possible graph of f .

Handwritten notes:
 Abs Max @ $x=0$ Abs Min @ $x=4$
 inf pt @ $x=2$

Ex1. Given the function

$$f(x) = 2x^3 + 3x^2 - 12x + 6$$

a.) Find the critical points.

$$f'(x) = 6x^2 + 6x - 12$$

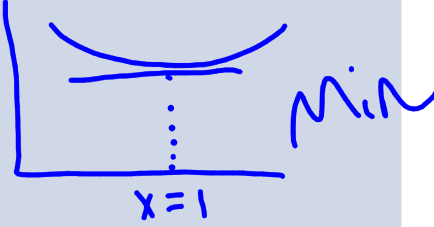
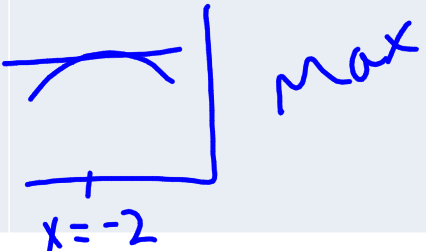
$$0 = 6(x^2 + x - 2)$$

$$0 = (x + 2)(x - 1)$$

$$x = -2, 1$$

b.) Find the second derivative at each of these points.

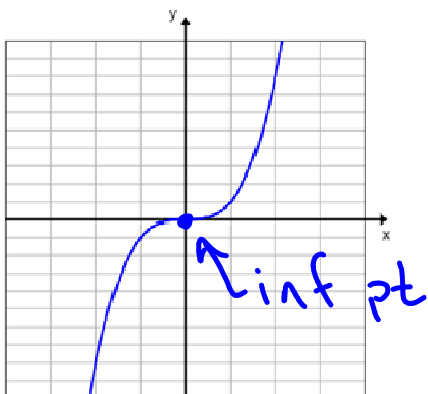
$$f' = 6x^2 + 6x - 12 \quad f'' = 12x + 6$$

x	f'	f''	Max/Min
<u>x=1</u>	0	+18	 A hand-drawn coordinate system showing a parabola opening upwards. A vertical dashed line drops from the vertex to the x-axis, which is labeled 'x=1'. The word 'Min' is written to the right of the graph.
x=-2	0	-18	 A hand-drawn coordinate system showing a parabola opening downwards. A vertical dashed line drops from the vertex to the x-axis, which is labeled 'x=-2'. The word 'Max' is written to the right of the graph.

The 2nd Derivative Test for Classifying Local Extrema

- If $f'(c) = 0$ and $f''(c) > 0$, then $x = c$ is a local minimum.
- If $f'(c) = 0$ and $f''(c) < 0$, then $x = c$ is a local maximum.

However, if $f'(c) = 0$ and $f''(c) = 0$, then the 2nd der test cannot be used to classify that extrema.

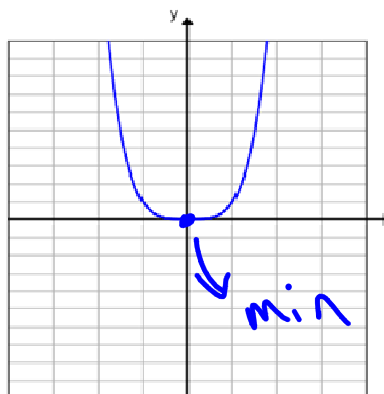


$$y = x^3$$

$$y' = 3x^2 \quad @x = 0, \quad y' = 0$$

$$y'' = 6x \quad @x = 0, \quad y'' = 0$$

Inflection Point



$$y = x^4$$

$$y' = 4x^3 \quad @x = 0, \quad y' = 0$$

$$y'' = 12x^2 \quad @x = 0, \quad y'' = 0$$

Minimum

If $f'(c) = 0$ and $f''(c) = 0$, the Second Derivative Test is inconclusive.

Ex2. Find and classify the extrema

$$f(x) = 2\cos(x) + \cos(2x) \quad \text{on} \quad 0 \leq x \leq 2\pi$$

Homework

pg 215 # 25-30, 33, 36-40, 48, 51,
52, 55-59